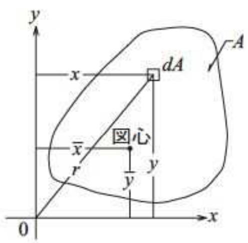
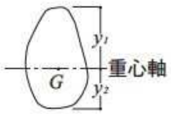
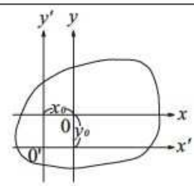
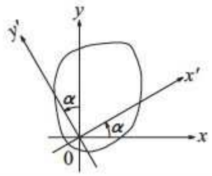
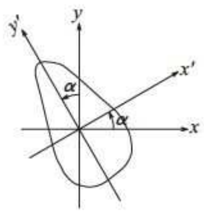
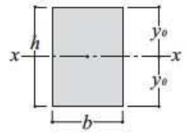
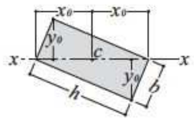
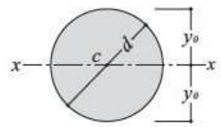
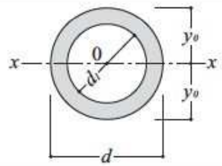
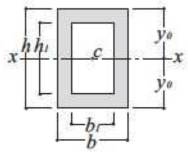
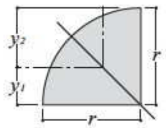
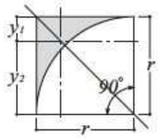
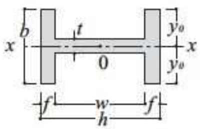
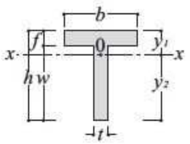
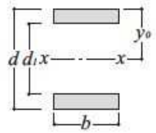
3-1. 平面図形

断面の諸係数の定義

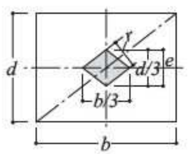
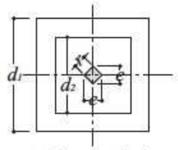
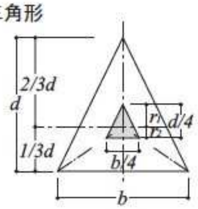
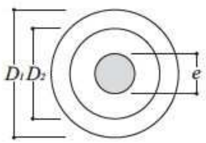
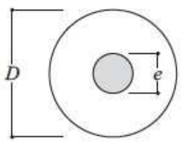
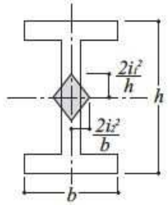
断面積	$A = \int_A dA$	
断面1次モーメント	$S_x = \int_A y dA$, $S_y = \int_A x dA$	
図心	$\bar{y} = \frac{S_x}{A}$, $\bar{x} = \frac{S_y}{A}$	
断面2次モーメント	$I_x = \int_A y^2 dA$, $I_y = \int_A x^2 dA$	
断面相乗モーメント	$I_{xy} = \int_A xy dA$	
断面極2次モーメント	$I_p = \int_A r^2 dA$, $I_p = I_x + I_y$	
断面2次半径	$i_x = \sqrt{\frac{I_x}{A}}$, $i_y = \sqrt{\frac{I_y}{A}}$	
断面極2次半径	$i_p = \sqrt{\frac{I_p}{A}}$	
断面係数	$Z_1 = \frac{I}{y_1}$, $Z_2 = \frac{I}{y_2}$	
軸の平行移動	$S_{x'} = S_x + Ay_0$, $S_{y'} = S_y + Ax_0$ $I_{x'} = I_x + 2y_0S_x + Ay_0^2$, $I_{y'} = I_y + 2x_0S_y + Ax_0^2$ $I_{x'y'} = I_{xy} + x_0S_x + y_0S_y + Ax_0y_0$ 0を図心にとれば $I_{x'} = I_x + Ay_0^2$, $I_{y'} = I_y + Ax_0^2$ $I_{x'y'} = I_{xy} + Ax_0y_0$	
軸の回転移動	$S_{x'} = S_x\cos\alpha - S_y\sin\alpha$ $S_{y'} = S_x\sin\alpha + S_y\cos\alpha$ $I_{x'} = I_x\cos^2\alpha + I_y\sin^2\alpha - I_{xy}\sin2\alpha$ $I_{y'} = I_x\sin^2\alpha + I_y\cos^2\alpha + I_{xy}\sin2\alpha$ $I_{x'y'} = \frac{I_x-I_y}{2}\sin2\alpha + I_{xy}\cos2\alpha$	
主軸および主断面2次モーメント	$I_{x'y'} = 0$となるとき、x', y' を主軸という 主軸の傾き $\tan2\alpha = \frac{2I_{xy}}{I_y-I_x}$ $I_{x'} = \frac{1}{2}(I_x+I_y) + \frac{1}{2}\sqrt{(I_x-I_y)^2+4I_{xy}^2}$ $I_{y'} = \frac{1}{2}(I_x+I_y) - \frac{1}{2}\sqrt{(I_x-I_y)^2+4I_{xy}^2}$ $I_{x'} = I_x\cos^2\alpha + I_y\sin^2\alpha$ $I_{y'} = I_x\sin^2\alpha + I_y\cos^2\alpha$ $I_{x'y'} = \frac{I_x-I_y}{2}\sin2\alpha$	

断面性能算出公式

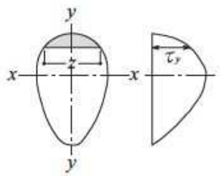
断　面	面　積 A	図心より縁に至る距離 y_0	断面2次モーメント I_x	断面2次半径 i_x	断面係数 Z_x
	bh	$\frac{h}{2}$	$\frac{bh^3}{12}$	$\frac{h}{\sqrt{12}}$ $=0.289h$	$\frac{bh^2}{6}$
	bh	$y_0 = \frac{bh}{\sqrt{b^2+h^2}}$ $x_0 = \frac{1}{2}\sqrt{b^2+h^2}$	$\frac{b^3h^3}{6(b^2+h^2)}$	$\frac{bh}{\sqrt{6(b^2+h^2)}}$	$\frac{b^2h^2}{6\sqrt{b^2+h^2}}$
	$\frac{\pi}{4}d^2$ $=0.785d^2$	$\frac{d}{2}$	$\frac{\pi}{64}d^4$ $=0.0491d^4$	$\frac{d}{4}$	$\frac{\pi}{32}d^3$ $=0.0982d^3$
	$\frac{\pi}{4}(d^2-d_1^2)$ $=0.785$ $\cdot(d^2-d_1^2)$	$\frac{d}{2}$	$\frac{\pi}{64}(d^4-d_1^4)$ $=0.0491$ $\cdot(d^4-d_1^4)$	$\frac{\sqrt{d^2+d_1^2}}{4}$	$\frac{\pi}{32}\frac{d^4-d_1^4}{d}$ $=0.0982$ $\cdot\frac{d^4-d_1^4}{d}$
	$bh-b_1h_1$	$\frac{h}{2}$	$\frac{bh^3-b_1h_1^3}{12}$	$\sqrt{\frac{bh^3-b_1h_1^3}{12(bh-b_1h_1)}}$	$\frac{bh^3-b_1h_1^3}{6h}$
	$\frac{\pi}{4}r^2$	$y_1=0.4244r$ $y_2=0.5756r$	$0.055r^4$	$0.2643r$	$Z_1=0.1296r^3$ $Z_2=0.0956r^3$
	$0.2146r^2$	$y_1=0.2234r$ $y_2=0.7766r$	$0.0075r^4$	$0.187r$	$Z_1=0.03357r^3$ $Z_2=0.00966r^3$

断　　面	面　積 A	図心より縁に至る距離 y_0	断面2次モーメント I_x	断面2次半径 i_x	断面係数 Z_x
	$bh-w(b-t)$	$\frac{b}{2}$	$\frac{2fb^3+wt^3}{12}$	$\sqrt{\frac{2fb^3+wt^3}{12\{bh-w(b-t)\}}}$	$\frac{2fb^3+wt^3}{6b}$
	$bf+wt$	$y_1=\frac{th^2+f^2(b-t)}{2(bf+wt)}$ $y_2=h-y_1$	$\frac{th^3+(b-t)f^3}{3}-Ay_1^2$	$\sqrt{\frac{I_x}{A}}$	$Z_1=\frac{I_x}{y_1}$ $Z_2=\frac{I_x}{y_2}$
	$b(d-d_1)$	$\frac{d}{2}$	$\frac{b(d^3-d_1^3)}{12}$	$\sqrt{\frac{d^3-d_1^3}{12(d-d_1)}}$ $=0.289\sqrt{d^2-dd_1+d_1^2}$	$\frac{b(d^3-d_1^3)}{6d}$

断面の核

矩形	中空正方形	三角形
 $e=d/3$ $r=\frac{bd}{3\sqrt{b^2+d^2}}$	 $e=\frac{11}{3}\left\{1+\left(\frac{d_2}{d_1}\right)^2\right\}$ $r=0.2358d_1\left\{1+\left(\frac{d_2}{d_1}\right)^2\right\}$	 $e=d/4$ $r_1=d/6$ $r_2=d/12$
中空円	**円**	**I形**
 $e=\frac{D_1}{4}\left\{1+\left(\frac{D_2}{D_1}\right)^2\right\}$	 $e=D/4$	

せん断応力



一般に　$\tau_y=\frac{QS_x}{bI_x}$

ここに

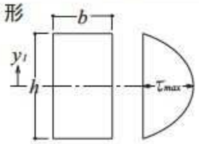
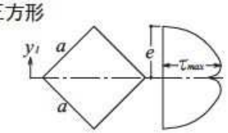
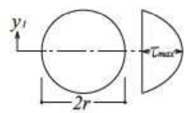
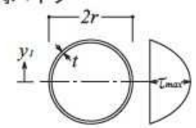
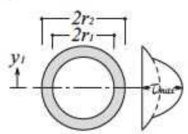
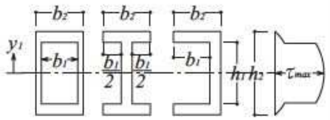
S_x：ハッチ部分の断面1次モーメント

I_x：断面2次モーメント

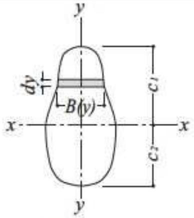
$\tau_{max}=\kappa\frac{Q}{A}$：最大せん断応力度

$\kappa=\frac{最大応力度}{平均応力度}$

y_1 ＝ 中立軸からの距離

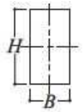
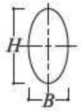
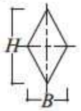
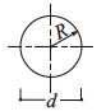
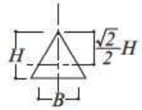
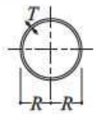
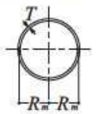
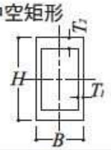
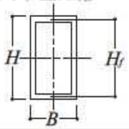
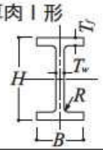
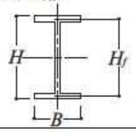
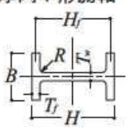
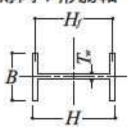
断　面　形	せん断力　τ	κ
1.矩　形 	$\tau=\frac{3}{2}\frac{Q}{bh}\left\{1-\left(\frac{2y_1}{h}\right)^2\right\}$ $\tau_{max}=\frac{3}{2}\frac{Q}{bh}=\frac{3}{2}\frac{Q}{A}$ $(y_1=0)$	$\frac{3}{2}$
2.正方形 	$\tau=\frac{Q}{a^2}\left\{1+\sqrt{2}\frac{y_1}{a}-4\left(\frac{y_1}{a}\right)^2\right\}$ $\tau_{max}=\frac{9}{8}\frac{Q}{a^2}=1.125\frac{Q}{A}$ $(y_1=\frac{1}{4}e=\frac{\sqrt{2}}{8}a)$	$\frac{9}{8}$
3.円 	$\tau=\frac{4}{3}\frac{Q}{\pi r^2}\left\{1-\left(\frac{y_1}{r}\right)^2\right\}$ $\tau_{max}=\frac{4}{3}\frac{Q}{\pi r^2}=\frac{4}{3}\frac{Q}{A}$ $(y_1=0)$	$\frac{4}{3}$
4.薄肉パイプ 	$\tau=\frac{Q}{\pi rt}\left\{1-\left(\frac{y_1}{r}\right)^2\right\}$ $\tau_{max}=\frac{Q}{\pi rt}=2\frac{Q}{A}$ $(y_1=0)$	2
6.パイプ 	$r_2\geqq y_1\geqq r_1$: $\tau=\frac{4}{3}\frac{Q}{\pi(r_2^4-r_1^4)}(r_2^2-y_1^2)$ $r_1\geqq y_1$: $\tau=\frac{4}{3}\frac{Q}{\pi(r_2^4-r_1^4)}\left\{r_2^2+r_1^2-2y_1^2+\sqrt{(r_2^2-y_1^2)(r_1^2-y_1^2)}\right\}$	$\kappa=\frac{4(r_2^2+r_2r_1+r_1^2)}{3(r_2^2+r_1^2)}$ $\tau_{max}=\frac{4}{3}\frac{Q(r_2^2+r_2r_1+r_1^2)}{\pi(r_2^4-r_1^4)}$ $=\frac{Q}{A}\frac{4(r_2^2+r_2r_1+r_1^2)}{3(r_2^2+r_1^2)}$
8.対称形断面 	$\frac{h_2}{2}\geqq y_1\geqq\frac{h_1}{2}$: $\tau=\frac{3Q}{2(b_2h_2^3-b_1h_1^3)}(h_2^2-4y_1^2)$ $\frac{h_1}{2}\geqq y_1$: $\tau=\frac{3Q}{2(b_2h_2^3-b_1h_1^3)}\left(\frac{b_2h_2^2-b_1h_1^2}{b_2-b_1}-4y_1^2\right)$	$\kappa=\frac{3(b_2h_2^2-b_1h_1^2)(b_2h_2-b_1h_1)}{2(b_2h_2^3-b_1h_1^3)(b_2-b_1)}$ $\tau_{max}=\frac{b_2h_2^2-b_1h_1^2}{(b_2h_2^3-b_1h_1^3)(b_2-b_1)}\frac{3Q}{2}$

塑性断面係数



$$Z_p = \int_0^{c_1} B(y)y\,dy + \int_0^{c_2} B(y)y\,dy$$

$B(y)$：中立軸から距離 y だけ離れたところの断面幅
c_1, c_2：それぞれ中立軸から上下縁までの距離

断　面　形	塑性断面係数　Z_p	断　面　形	塑性断面係数　Z_p
矩形 	$B \cdot H^2/4$	楕円 	$B \cdot H^2/6$
菱形 	$B \cdot H^2/12$	斜正方形 	$\sqrt{2}\,a^3/6$
円 	$d^3/6,\ \frac{4}{3}R^3$	正三角形 	$\frac{2-\sqrt{2}}{6}B \cdot H^2$
中空円 	$\frac{4}{3}R^3\{1-(1-\frac{T}{R})^3\}$	薄肉中空円 	$4 \cdot R_m{}^2 \cdot T$
中空矩形 	$B \cdot T_2(H-T_2)$ $+\frac{1}{2}(H-2T_2)^2 T_1$	薄肉中空矩形 	$A_f \cdot H_f + \frac{1}{4}A_w \cdot H_f$ A_f：片側フランジプレートの断面積 A_w：ウェブプレートの全断面積
厚肉 I 形 	$B \cdot T_f(H-T_f)$ $+\frac{1}{4}(H-2T_f)^2 T_w$ $+0.4292R^2(H-2T_f-0.4467R)$	薄肉 I 形 	$A_f \cdot H_f + \frac{1}{4}A_w \cdot H_f$ A_f：片側フランジの断面積 A_w：ウェブの全断面積
厚肉 I 形弱軸 	$\frac{1}{2}B^2 \cdot T_f + \frac{1}{4}(H-2T_f) \cdot T_w{}^2$ $+0.4292R^2(T_w+0.4467R)$	薄肉 I 形弱軸 	$\frac{1}{2}A_f \cdot B + \frac{1}{4}A_w \cdot T_w$ A_f：片側フランジの断面積 A_w：ウェブの全断面積